"On the Viscosity of Argon as affected by Temperature." By
    LORD RAYLEIGH, F.R.S. Received January 12,—Read Jan-
    uary 18, 1900.

According to the kinetic theory, as developed by Maxwell, the
viscosity of a gas is independent of its density, whatever may be the
character of the encounters taking place between the molecules.   In
the typical case of a gas subject to a uniform shearing motion, we may
suppose that of the three component velocities v and w vanish, while u
is a linear function of y, independent of x and z.   If μ be the viscosity,
the force transmitted tangentially across unit of area perpendicular to
y is measured by $\mu\,du/dy$.   This represents the relative momentum,
parallel to x, which in unit of time crosses the area in one direction,
the area being supposed to move with the velocity of the fluid at the
place in question.   We may suppose, for the sake of simplicity, and
without real loss of generality, that u is zero at the plane.   The
momentum, which may now be reckoned absolutely, does not vanish,
as in the case of a gas at rest throughout, because the molecules come
from a greater or less distance, where (*e.g.*) the value of u is positive.
The distance from which (upon the average) the molecules may be
supposed to have come depends upon circumstances.   If, for example,
the molecules, retaining their number and velocity, interfere less with
each other's motion, the distance in question will be increased.   The
same effect will be produced, without a change of quality, by a simple
reduction in the number of molecules, *i.e.*, in the density of the gas,
and it is not difficult to recognise that the distance from which the
molecules may be supposed to have come is *inversely as the density*.   On
this account the passage of tangential momentum *per molecule* is in-
versely as the density, and since the number of molecules crossing is
directly as the density, the two effects compensate, and upon the
whole the tangential force and therefore the viscosity remain un-
altered by a change of density.

On the other hand, the manner in which this viscosity varies with
temperature depends upon the nature of the encounters.   If the
molecules behave like Boscovich points, which exercise no force upon
one another until the distance falls to a certain value, and which then
repel one another infinitely (erroneously called the theory of elastic
spheres), then, as Maxwell proved, the viscosity would be proportional
to the square root of the absolute temperature.   Or again, if the
law of repulsion were as the inverse fifth power of the distance,
viscosity would be as the absolute temperature.

In the more general case where the repulsive force varies as r^{-n}, the
dependence of μ upon temperature may also be given.   If v be the
velocity of mean square, proportional to the square root of the tem-

perature, μ varies as $v^{\frac{n+3}{n-1}}$, a formula which includes the cases $(n=5,$ $n=\infty)$ already specified. If we assume the law already discussed —that μ is independent of density—this conclusion may be arrived at very simply by the method of "dimensions."

In order to see this we note that the only quantities (besides the density) on which μ can depend are m the mass of a particle, v the velocity of mean square, and k the repulsive force at unit distance. The dimensions of these quantities are as follows :—

$$\mu = (\text{mass})^1 (\text{length})^{-1} (\text{time})^{-1},$$

$$m = (\text{mass})^1,$$

$$v = (\text{length})^1 (\text{time})^{-1},$$

$$k = (\text{mass})^1 (\text{length})^{n+1} (\text{time})^{-2}.$$

Thus, if we assume

$$\mu \propto m^x \cdot v^y \cdot k^z \dots\dots\dots\dots\dots\dots\dots \quad (1),$$

we have $\quad 1 = x+z, \quad -1 = y+(n+1)z, \quad -1 = -y-2z,$

whence $\qquad x = \dfrac{n+1}{n-1}, \qquad y = \dfrac{n+3}{n-1}, \qquad z = \dfrac{2}{n-1}.$

Accordingly $\qquad \mu = \alpha \cdot m^{\frac{n+1}{n-1}} \cdot v^{\frac{n+3}{n-1}} \cdot k^{-\frac{2}{n-1}} \dots\dots\dots\dots\dots\dots \quad (2),$

where α is a purely numerical coefficient. For a given kind of molecule, m and k are constant. Thus

$$\mu \propto v^{\frac{n+3}{n-1}} \propto \theta^{\frac{n+3}{2n-2}} \dots\dots\dots\dots\dots\dots\dots \quad (3).$$

The case of sudden impacts $(n=\infty)$ gives, as already remarked, $\mu \propto v \propto \theta^{\frac{1}{2}}$. Hence k disappears, and the consideration of dimensions shows that $\mu \propto d^{-2}$, where d is the diameter of the particles.

The best experiments on air show that, so far as a formula of this kind can represent the facts, $\mu \propto \theta^{0 \cdot 77}$. It may be observed that $n = 8$ corresponds to $\mu \propto \theta^{0 \cdot 79}$.

When we remember that the principal gases, such as oxygen, hydrogen, and nitrogen, are regarded as diatomic, we may be inclined to attribute the want of simplicity in the law connecting viscosity and temperature to the complication introduced by the want of symmetry in the molecules and consequent diversities of presentation in an encounter. It was with this idea that I thought it would be interesting to examine the influence of temperature upon the viscosity of argon, which in the matter of specific heat behaves as if composed of

single atoms. From the fact that no appreciable part of the total energy is rotatory, we may infer that the forces called into play during our encounter are of a symmetrical character. It seemed, therefore, more likely that a simple relation between viscosity and temperature would obtain in the case of argon than in the case of the "diatomic" gases.

The best experimental arrangement for examining this question is probably that of Holman,* in which the same constant stream of gas passes in succession through two capillaries at different temperatures, the pressures being determined before the first and after the second passage, as well as between the two. But to a gas like argon, available in small quantities only, the application of this method is difficult. And it seemed unnecessary to insist upon the use of constant pressures, seeing that it was not proposed to investigate experimentally the dependence of transpiration upon pressure.

The theoretical formula for the volume of gas transpired, analogous to that first given by Stokes for an incompressible fluid, was developed by O. E. Meyer.† Although not quite rigorous, it probably suffices for the purpose in hand. If p_1, V_1 denote the pressure and volume of the gas as it enters the capillary, p_2, V_2 as it leaves the capillary, we have

$$p_1 V_1 = p_2 V_2 = \frac{\pi t \mathrm{R}^4}{16\,\mu l}\left(p_1{}^2 - p_2{}^2\right) \quad \ldots\ldots\ldots\ldots \quad (4).$$

In this equation t denotes the time of transpiration, R the radius of the tube, l its length, and μ the viscosity measured in the usual way.

In order to understand the application of the formula for our present purpose, it will be simplest to consider first the passage of equal volumes of different gases through the capillary, the initial pressures, and the constant temperature being the same. In an apparatus, such as that about to be described, the pressures change as the gas flows, but if the pressures are definite functions of the amount of gas which at any moment has passed the capillary, this variation does not interfere with the proportionality between t and μ. For example, if the viscosity be doubled, the flow takes place precisely as before, except that the scale of time is doubled. It will take twice as long as before to pass the same quantity of gas.

Although different gases have been employed in the present experiments, there has been no attempt to compare their viscosities, and indeed such a comparison would be difficult to carry out by this method. The question has been, how is the viscosity of a given gas affected by a change of temperature? In one set of experiments the

* ' Phil. Mag. ' vol. 3, p. 81, 1877.

† ' Pogg. Ann.,' vol. 127, p. 269, 1866.

capillary is at the temperature of the room; in a closely following set the capillary is bathed in saturated steam at a temperature that can be calculated from the height of the barometer.

If the temperature were changed throughout the whole apparatus from one absolute temperature θ to another absolute temperature θ', we could make immediate application of (4); the viscosities (μ, μ') at the two temperatures would be directly as the times of transpiration (t, t'). The matter is not quite so simple when, as in these experiments, the change of temperature takes place only in the capillary. A rise of temperature in the capillary now acts in two ways. Not only does it change the viscosity, but it increases the *volume* of gas which has to pass. The ratio of volumes is θ', θ; and thus

$$\frac{\mu'}{\mu} = \frac{t'}{t} \times \frac{\theta}{\theta'} \quad \dots\dots\dots\dots\dots\dots\dots \quad (5),$$

subject to a small correction for the effect of temperature upon the dimensions of the capillary. It is assumed that the temperature of the reservoirs is the same in both transpirations.

The apparatus is shown fig. 1. The gas flows to and fro between the bulbs A and B, the flow from A to B only being timed. It is confined by mercury, which can pass through U connections of blown glass from A to C and from B to D. The bulbs B, C, D are supported upon their seats with a little plaster of Paris. The capillary is nearly 5 feet (150 cm.) in length and is connected with the bulbs by gas tubing of moderate diameter, all joints being blown. E represents the jacket through which steam can be passed; its length exceeds that of the capillary by a few inches.

In order to charge the apparatus, the first step is the exhaustion. This is effected through the tap, F, with the aid of a Toppler pump, and it is necessary to make a corresponding exhaustion in C and D, or the mercury would be drawn over. To this end the rubber terminal H is temporarily connected with G, while I leads to a common air pump. When the exhaustion is complete, the gas to be tried is admitted gradually at F, the atmosphere being allowed again to exert its pressure in C and D. When the charge is sufficient, F is turned off, after which G remains open to the atmosphere, and H is connected to a manometer.

When a measurement is commenced, the first step is to read the temperatures of the bulbs and of the capillary; I is then connected to a force pump, and pressure is applied until so much of the gas is driven over that the mercury below A and in B assumes the positions shown in the diagram. I is then suddenly released so that the atmospheric pressure asserts itself in D, and the gas begins to flow back into B. The bulb J allows the flow a short time in which to establish itself before the time measurement begins as the mercury passes the

connection passage K. When the mercury reaches L, the time measurement is closed.

One of the points to be kept in view in designing the apparatus is to secure long enough time of transpiration without unduly lowering the driving pressure. At the beginning of the measured transpiration the pressure in A was about 30 cm. of mercury above atmosphere, and

Fig. 1.
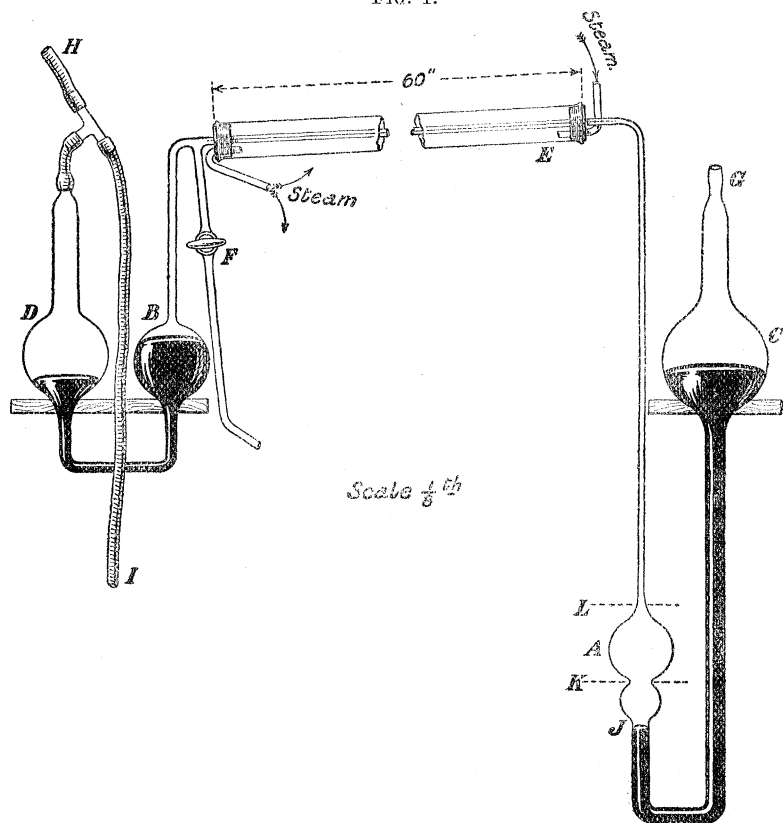
that in B about 2 cm. below atmosphere. At the end the pressure in A was 20 cm., and in B 3 cm., both above atmosphere. Accordingly the driving pressure fell from 32 to 17 cm.

Three, or, in the case of hydrogen, five, observations of the time were usually taken, and the agreement was such as to indicate that the mean would be correct to perhaps one-tenth of a second. The time for air at the temperature of the room was about ninety seconds, and for hydrogen forty-four seconds, but these numbers are not strictly comparable.

When the low temperature observations were finished, the gas was lighted under a small boiler placed upon a shelf above the apparatus, and steam was passed through the jacket. It was necessary to see that there was enough heat to maintain a steady issue of steam, yet not so much as to risk a sensible back pressure in the jacket. The time of transpiration for air was now about 139 seconds. Care was always taken to maintain the temperature of the bulbs at the same point as in the first observations.

There are one or two matters as to which an apparatus on these lines is necessarily somewhat imperfect. In the high temperature measurements the whole of the gas in the capillary is assumed to be at the temperature of boiling water, and all that is not in the capillary to be at the temperature of the room, assumptions not strictly compatible. The compromise adopted was to enclose in the jacket the whole of the capillary and about 2 inches at each end of the approaches, and seems sufficient to exclude sensible error when we remember the rapidity with which heat is conducted in small spaces. A second weak point is the assumption that the instantaneous pressures are represented by the heights of the moving mercury columns. If the connecting U-tubes are too narrow, the resistance to the flow of mercury enters into the question in much the same way as the flow of gas in the capillary. In order to obtain a check upon this source of error the apparatus has been varied. In an earlier form the connecting U-tubes were comparatively narrow ; but the result for the ratio of viscosities of hot and cold air was substantially the same as that subsequently obtained with the improved apparatus, in which these tubes were much widened. Even if there be a sensible residual error arising from this cause, it can hardly affect the comparison of temperature-coefficients of gases whose viscosity is nearly the same.

I will now give an example in detail from the observations of December 21 with purified argon. The times of transpiration at the temperature of the room (15° C.) were in seconds

$$104\tfrac{3}{4}, \quad 104\tfrac{1}{2}, \quad 104\tfrac{3}{4}. \quad \text{Mean, } 104\cdot67.$$

When the capillaries were bathed in steam, the corresponding times were

$$167\tfrac{1}{2}, \quad 167\tfrac{1}{2}, \quad 167\tfrac{3}{4}. \quad \text{Mean, } 167\cdot58.$$

The barometer reading (corrected) being 767·4 mm., we deduce as the temperature of the jacket 100·27° C. Thus $\theta = 287\cdot5$, $\theta' = 372\cdot8$. The reduction was effected by assuming

$$\frac{t'}{t} = \left(\frac{\theta'}{\theta}\right)^x \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (6).$$

With the above values we get

$$x = 1\cdot812.$$

As appears from (5), the integral part of x relates merely to the expansion of the gas by temperature. If we take

$$\frac{\mu'}{\mu} = \left(\frac{\theta'}{\theta}\right)^n \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (7),$$

we get $n = 0\cdot812.$

This number is, however, subject to a small correction for the expansion of the glass of the capillary. As appears from (4), the ratio μ', μ as used above requires to be altered in the same ratio as that in which the glass expands by volume. The value of n must accordingly be increased by $0\cdot010$, making

$$n = 0\cdot822.$$

The following table embodies the results obtained in a somewhat extended series of observations. The numbers given are the values of n in (7), corrected for the expansion of the glass.

Air (dry)	$0\cdot754$
Oxygen	$0\cdot782$
Hydrogen	$0\cdot681$
Argon (impure)............	$0\cdot801$
Argon (best)	$0\cdot815$

In the last trials, the argon was probably within 1 or 2 per cent. of absolute purity. The nitrogen lines could no longer be seen, and scarcely any further contraction could be effected on sparking with oxygen or hydrogen.

It will be seen that the temperature change of viscosity in argon does not differ very greatly from the corresponding change in air and oxygen. At any rate the simpler conditions under which we may suppose the collisions to occur, do not lead to values of n such as $0\cdot5$, or $1\cdot0$, discussed by theoretical writers.

I may recall that, on a former occasion,* I found the viscosity of argon to be $1\cdot21$ relatively to that of air, both being observed at the temperature of the room.

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* 'Roy. Soc. Proc.,' January, 1896.

FIG. 1.

Scale ⅛ᵗʰ